Issues of and research on routine problem solving

Research evidence indicates that children tend to be good at routine problem solving if they understand the meanings of the arithmetic operations and the concept of ratio well. While there are only four basic arithmetic operations, there are more than four distinct meanings for these operations. How well are schools doing with respect to teaching routine problem solving? The Toys "R" Us anecdote and the shepherd research example suggest that the answer may be ‘not well’. There is much research evidence that also suggests 'not well'. Here is a example that concerns former students (pre-service teachers) of mine.

Two pre-service teachers gave the following problem to some grades 4 and 5 students (Cesmystruk & Zirk, 1995).

> **Harry went on a trip to the Nile River. His task was to completely fill his crocodile egg incubator. On the first day of his journey, Harry came upon a crocodile nest. He dug up 23 unhatched crocodile eggs and placed them in the incubator. Harry walked 5 paces and dug up some more unhatched crocodile eggs which he quickly placed in the incubator as well. Harry’s task was now done as he had completely filled his egg incubator with 40 unhatched crocodile eggs. How many eggs did Harry find at the second crocodile nest?**

The story problem is about ‘put together’ and that is modeled mathematically by addition. The mathematical structure of the problem can be represented by the number sentence; ’23 + ? = 40'. The students that tried to solve the crocodile egg problem had been taught two problem-solving strategies: guess and check and look for a pattern. The following are edited transcriptions of the responses of three of the students. The responses are typical of the thinking of the other students.

Student #1

The student read the problem carefully, underlined all the printed numbers, and then said “*Easy numbers!*”. The answer that the student gave (44) was incorrect (but strongly insisted that it was correct). The student’s work consisted of three steps: (1) 40 + 5 = 45, (2) 45 - 23 = 22, and (3) 22 x 2 = 44.

To explain the work, the student stated that since there were forty eggs, it was necessary to add the little number (the ‘5’), and then to subtract 23. That answer had to be multiplied by 2 because the problem had stated the word ‘second’ somewhere.

Student #2

The student did not do the problem. The student explained that the problem did not make any sense because it did not fit either of the problem-solving strategies learned in class.

Student #3

> The student made a guess and check chart, but did not do any guessing. The student obtained the correct solution (17) by subtracting 23 from 40. The student could not explain the subtraction strategy. When asked if any guessing was done, the reply was "*No*." When asked why the guess chart was made, the student said; "*All word problems are done by guess and check.*"

The three student responses above illustrate several things, one of which concerns the inadequacy of the strategies 'guess and check' and 'look for a pattern' for solving problems like the one about the crocodile eggs. Those strategies have little value in relation to routine problem solving. They can, however, be quite useful for non-routine problem solving.

Is the anecdote an indication of what may generally be the case with respect to proficiency in routine problem solving? The short answer is 'yes'. Assessment results tend to find that children and adults are not proficient at routine problem solving (see for example, Rudnitsky et al, 1995).

An explanation for the assessment results can be found in the strategies that children use for routine problem solving. These strategies reflect what is explicitly taught by teachers or they reflect what children construct from what is taught or not taught as they try to make sense of their classroom experiences. The strategies that children often use take a variety of forms. The following is a partial list of them (Sowder, 1988):

- Find the numbers and add (or multiply or . . .). The choice tends to be dictated by recent activities or by what the child feels comfortable with.
- Guess the operation to be used and see what you get. This is a form of 'guess and check'.
- Look at the numbers; they will "tell" you which operation to use (for example, the numbers 63 and 59 suggest add or subtract, while 25 and 5 suggest divide).
- Try all the operations and select the most reasonable answer.
- Look for key words which tell you what to do.
- Decide whether the answer should be larger or smaller than the given numbers. If larger, try both addition and multiplication, and select the more reasonable answer. If smaller, try both subtraction and division and select the more reasonable answer.
- Choose the operation whose meaning fits what is happening in the problem.

Of the listed strategies, only the last one can readily be applied to more than one-step word problems (or to real life problems). That strategy concerns choosing the operation whose meaning fits the story. It can be interpreted as identifying what is going on in the problem (finding the structure) and then representing the structure of the event or situation in a mathematical way. Mathematical forms such as number sentences can be used for that purpose. The problem about the crocodile eggs discussed earlier illustrates the approach. For that problem, the story was about combining eggs in a 'put together' sense and that can be modeled mathematically by the number sentence, '23 + ? = 40'. This number sentence can then be altered into one that is more convenient for doing arithmetic (e. g. 40 - 23 = ?).

Research has shown that one of the characteristics of good problem solvers is that they seek the structure of the problem rather than searching for the algorithm to apply for obtaining the answer (see for example, Hembree, 1992). That research strongly suggests that, for purposes of developing proficiency in routine problem solving, it is preferable that teachers use an approach that involves modeling/representing in some way the events or circumstances of problems.

For example, a study found that modeling is a useful strategy for routine problem solving as early as Kindergarten (Carpenter et al, 1993). The Kindergarten children in the study had spent a year solving a variety of basic oral word problems related to addition, subtraction, multiplication, and division (yes!). They used concrete materials to model the events of the stories (no number sentences). Overall, the children demonstrated a remarkable ability in solving the problems. They did as well as or better than children in grades 1 and 3 who solved similar problems but who did not consistently use modeling as the problem-solving strategy. The Kindergarten children's success in solving the word problems can be explained by their consistent use of modeling to represent the actions of the problems in a way that reflected what was going on in the story.

The Kindergarten study provides strong evidence that seeking the structure and modeling the event or action of the problem can be a powerful strategy for routine problem solving that involves the meanings of the arithmetic operations. The meanings of the arithmetic operations are formal versions of the concrete models used by the Kindergarten children for solving the problems. Unfortunately, typical teaching practice does not often develop the strategy of identifying and representing the structure of the event or action of the problem.

Typical teaching practice tends to develop less empowering strategies. Children are sometimes taught to match the given problem to a particular arithmetic operation. [For example, "For these kinds of problems, you always add to get the answer."] This labeling approach tends not to work well as soon as the child encounters problems that require a different arithmetic operation or that require more than one arithmetic operation. In other words, if children are given a mixture of problems to solve they are in trouble because they have only learned to solve problems that have been pre-labeled with what arithmetic to do.

Some teachers teach children to use strategies such as read the problem carefully and underline the numbers. This is hardly helpful if the child does not understand what is going on in the problem. A popular teaching approach involves key words. For example, if the word 'minus' appears in a problem, then you subtract to get the answer; if the word 'more' appears then you add to get the answer; and so on. This 'search for key words' strategy is inappropriate for at least two reasons.

- A search for key words has little to do with trying to understand what the problem is actually about. Therefore, transfer to more complex problems - those that contain more than one step or that are real - tends to be difficult. For complex problems, simply picking out key words as indicators of what to do often leads to incorrect solutions. In contrast to the problems that are typically found in mathematics text books and resource books, many real world problems do not contain key words. There rarely are any flashing neon lights (key words) that signal what to do to get the answer. For example, the following problem does not have any key words. It does not even contain an overt question (which is often the case in real world problems). Therefore the person who relies on key words is unlikely to solve problems such as the one below.

The mail order from the States includes 3 shirts at $12.75, 5 pants at $38.99, and 2 jackets at $95.95. The shipping charges are $21.38. The sales tax is 7% applied to the purchases only and the GST is 7% applied to everything. The money exchange rate is 31% in favour of the States. You pay for the order in full by cheque.

- Key words can be misleading. For example, teachers sometimes tell children that 'more' means you add. Actually, 'more' is a comparison word. When we say things like "I put 5 more eggs in the basket.", It is 'put' that indicates addition might be needed. The word 'more' is simply a short way of saying "more than there used to be in the basket". As well, 'more' can be misleading. Adding 127 and 15 in the following problem would lead to an interesting answer.

Johnny has 139 candies. He ate 12 of them. Now he has 127 candies. Then he ate 15 more candies. How many candies does Johnny have now?

All key words can be misleading. Even the word 'total' can be misleading. Is 'total' always a signal to add? Consider the following two problems.

- **Bucky the squirrel was very busy storing acorns for the winter. Bucky had a large family. He hid 2345 acorns in each of 9856 holes in the ground. What was the total number of acorns Bucky hid?**

- **Sparks kept all his money in a piggy bank. When he counted it one day he found that he had $345. He decided to put 2/3 of his money in a real bank. What was the total amount of money he put in the real bank?**

To appreciate the trap that keywords can be for students consider this example. A student quickly answered the following problem correctly by saying that the answer was 20%.

> **Four out of five coaches recommend heavy weight lifting as an important part of becoming an athlete. What percent of coaches do not recommend that?**

The student's teacher was curious how the student obtained the answer of 20% so quickly. The teacher asked the student to explain how the answer was figured out. The student replied: I multiplied 4 x 5 because 'of' means multiply.

The curricular goal is clear. Children should be able to do routine problem solving well. Children's abilities to solve those problems depends to a large degree on them having deep and clear multi-context understandings of the four arithmetic operations (and of ratio). That requires teaching practice that focuses on meaning and on structure, not on finding key words, underlining numbers, or guessing and checking. A good teaching strategy is to have children write/tell their own story problems. Research studies have found that to be the case (see for example, Rudnitsky et al, 1995). The findings suggest that students who create their own word problems have greater proficiency in routine problem solving and that it improves over time.

Some educators see using story (word) problems as somehow being "traditional" and therefore bad teaching practice. The issue needs to be addressed. Instructional practice that makes use of real situations to develop proficiency in routine problem solving has powerful potential. For example, having children learn through classroom store situations where they buy and sell things; through situations such as school fund raising; and through doing science experiments can be meaningful contexts for developing skills in routine problem solving. However, such situations have their limitations. It may not always be feasible to use them because they can be time-intensive and resource-costly. They do not necessarily allow for a great diversity of problem situations (and that is needed for developing proficiency in routine problem solving). Story problems do not have these limitations. For that reason alone, both teacher-provided and student-authored story problems should be part of an instructional plan. In the case of student-authored problems, that provides an opportunity as well for doing authentic writing in the context of learning mathematics.

The following story problem is a sample of what grades 3 and 4 students can author (Rudnitsky et al, 1995). The action in the story involves 'put together' which can be represented by the number sentence; '17 + ? = 46'.

> **Nate went to Bird's Store and bought 17 sourballs and some lollipops. When he got home his friend Mike was there and they counted the candy. There were 46 pieces of candy. How many lollipops did Nate buy?**

References

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